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Clusters of galaxies with modified Newtonian dynamics (MOND)

R.H. Sanders
Kapteyn Astronomical Institute, P.O. Box 800, 9700 AV Groningen, The Netherlands

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ABSTRACT

X-ray emitting clusters of galaxies are considered in the context of modified Newtonian dynamics (MOND). I show that self-gravitating isothermal gas spheres are not good representations of rich clusters with respect to the radial gas density distribution as indicated by the X-ray surface brightness. Pure gas spheres with a density distribution described by a β model, as observed, also fail because, with MOND, these objects are far from isothermal and have a gas mass and luminosity much larger than observed for clusters of the same mean temperature. These problems may be resolved by adding an additional dark mass component in the central regions; a constant density sphere contained within two core radii and having a total mass of one to two times that in the gas. With this additional component, the observed luminosity-temperature relation for clusters of galaxies is reproduced. When the observed X-ray surface brightness distribution in actual clusters is modeled by such a two-component structure, the typical mass discrepancy is three to four times smaller than with Newtonian dynamics. Thus while MOND significantly reduces the mass of the dark component in clusters it does not remove it completely. I speculate on the nature of the dark component and argue that this is not a fundamental problem for MOND.

Key words: galaxy clusters: X-ray emission: kinematics and dynamics– dark matter, gravitation

1 INTRODUCTION

The modified Newtonian dynamics (MOND) is an empirically-based modification of Newtonian gravity or inertia in the limit of low accelerations ($< a_o \approx cH_o$) suggested by Milgrom (1983) as an alternative to cosmic dark matter. In addition to explaining galaxy scaling relations (Tully-Fisher, Faber-Jackson, Fundamental Plane), this simple algorithm allows one to accurately predict the shapes of spiral galaxy rotation curves from the observed distribution of gaseous and stellar matter. MOND also accounts for the kinematics of small groups of galaxies (Milgrom 1998) and of superclusters, as exemplified by the Perseus-Pisces filament (Milgrom 1997) without the need for unseen mass. These well-documented phenomenological successes (Sanders & McGaugh 2002 and references therein) challenge the cold dark matter (CDM) paradigm and provide some support for the suggestion that the current theory of gravity and inertia (General Relativity) may need revision in the limit of low accelerations or field gradients.

However, problems do arise when one attempts to apply MOND to the large clusters of galaxies. The and White (1988) first noted that, to successfully account for the discrepancy between the observed mass and the traditional virial mass in the Coma Cluster, the MOND acceleration parameter, supposedly a universal constant, should be about a factor of four larger than the value implied by galaxy rotation curves. With MOND, the dynamical mass of a pressure supported system at temperature T is $M \propto T^2/a_o$; therefore, the The and White result could also be interpreted as an indication that the MOND dynamical mass is still larger than the detectable mass in stars and gas.

In astronomical tests involving an individual extragalactic object, such as the Coma cluster, a contradiction is not necessarily a falsification. One can always argue that the peculiar aspects of an object, such as deviations from spherical symmetry or incomplete dynamical relaxation, exempt that particular case. However, Gerbal et al. (1992), looking at a sample of eight X-ray emitting clusters, noted that the problem is more general: although MOND reduced the Newtonian discrepancy by a factor of 10, there is still a need for dark matter, particularly in the central regions. Later, considering a large sample of X-ray emitting clusters, I found that the mass predicted by MOND remains, typically, a factor of two or three times larger than the total mass observed in the hot gas and in the stellar content of the galaxies

(Sanders 1998). More recently, Aguirre, Schaye & Quataert (2001) pointed out that MOND is inconsistent with the observed temperature gradient in inner regions of three clusters for which such data is available. Again, the problem can be remedied by additional non-luminous mass, primarily in the inner regions, of order two or three times the observed gas mass. This discrepancy is also evident from strong gravitational lensing in the central regions of clusters– the formation of multiple images of background galaxies. Here, MOND does not apply because accelerations are Newtonian, and the implied surface density greatly exceeds that of visible matter and hot gas (Sanders 1998). So, although MOND clearly reduces the classical Newtonian mass discrepancy in clusters of galaxies, there still remains a missing mass problem, particularly in the cores.

Here I consider the issue of the remaining missing mass in clusters and whether or not this a fundamental problem for MOND. First I calculate the structure and X-ray surface brightness distribution of MOND isothermal gas spheres. Except for the very central regions, the structure of these objects is self-similar; the finite mass is primarily determined by the temperature and is very weakly dependent upon the central density. Thus there exists a mass-temperature relation ($M \propto T^2$) which is absolute; this implies a gas mass typically 5 to 10 times larger than that observed in X-ray emitting clusters of the same temperature. Moreover, for central electron densities in the range of 0.001 to 0.01, there is a well-defined X-ray luminosity-temperature relation which is less steep ($L \propto T^{1.5}$) and lies well above the observed luminosity-temperature relation for clusters ($L \propto T^{2.5}$). Looking at individual objects, the radial dependence of X-ray surface brightness does not reproduce that typically observed in X-ray emitting clusters– observations which are well-fit by the traditional "β-model" (Sarazin 1988). The conclusion is that self-gravitating MOND isothermal gas spheres are not good representations of clusters of galaxies.

Gas spheres with a density distribution described by a β-model are not isothermal in the context of MOND. A central or boundary temperature must be specified to determine the run of temperature in such objects, but those models with the lowest temperature gradients have large core radii and are again over-massive and over-luminous with respect to observed clusters of the same temperature. These problems may be solved by postulating the existence of a second rigid component in the mass distribution: a constant density core having a radius about twice that of the β model core and a central surface density comparable to a_o/G. Here, by rigid I mean a component with a fixed density distribution which does not respond to the gravitational field of the hot gas or galaxies. The presence this additional component in the Coma cluster is implied by the MOND hydrostatic gas equation for cumulative mass. If this component is generally present in clusters, it contributes to the gravitational force in the inner regions and reduces the core radius at a given temperature. In this way the observed luminosity-temperature and mass-temperature relations for clusters may be reproduced.

About 40 individual clusters have been considered in terms of such a two-component model; i.e., the observed surface-brightness distributions and mean temperatures are fit by specifying the density of the non-luminous component which is assumed to extend to two gas core radii. The total mass of this additional component varies between a few times 10^{12} and 10^{14} $M_\odot$ and the implied mass-to-light ratio is typically 50 in solar units. Therefore, the required rigid component is not a standard stellar population; as noted previously, MOND requires dark, or heretofore undetected, matter in the central regions of rich clusters. Although the total discrepancy between dark and detectable mass is reduced by, on average, a factor of four over that implied by Newtonian dynamics, it is clear that there remains a dark matter problem for MOND. I discuss the issue of whether or not this is a contradiction. I speculate on the possible nature of this non-luminous component and argue that neutrinos of finite mass are a possible candidate.

2 THE STRUCTURE AND PROPERTIES OF MOND GAS SPHERES

2.1 MOND isothermal spheres

The structure of isotropic isothermal spheres may be determined by solving the equation of hydrostatic equilibrium:

$$\frac{dp}{dr} = -\rho g \tag{1a}$$

with the pressure p given by

$$p = \rho {\sigma_r}^2 \tag{1b}$$

where σ_r is the one-dimensional velocity dispersion (constant for an isothermal sphere), ρ is the density. The gravitational acceleration, g, in the context of MOND, is given by

$$g\mu(g/a_o) = g_n. \tag{2a}$$

where g_n is the usual Newtonian gravitational acceleration,

$$g_n = \frac{GM(r)}{r^2}, \tag{2b}$$

a_o is the critical acceleration below which gravity deviates from Newtonian (found to be about 10^{-8} cm/s^{-2} from fits to galaxy rotation curves), and $\mu(x)$ is a function which interpolates between the Newtonian regime ($\mu(x) = 1$ when $x >> 1$) and the MOND regime ($\mu(x) = x$ when $x << 1$). A function having this asymptotic form,

$$\mu(x) = x(1 + x^2)^{-\frac{1}{2}}, \tag{2c}$$

works well for galaxy rotation curves and is also used here.

Milgrom (1984) demonstrated that MOND isothermal spheres have a finite mass and a density which falls of as $r^{-\alpha}$ where $\alpha \approx 4$ at large radii. In the outer regions, where $M(r) = M =$ constant, it follows immediately from eqs. 1 and 2 that

$${\sigma_r}^4 = \alpha^{-2} G M a_o \tag{3}$$

which means that the mass is uniquely determined by the specified velocity dispersion or temperature. For an isothermal gas sphere this relation becomes

$$M \approx \frac{16}{Ga_o}\Big(\frac{kT}{fm_p}\Big)^2 = (2.9 \times 10^{13}) {T_{keV}}^2 \, M_\odot \tag{4}$$

where f is the mean atomic weight ($\approx$ 0.62 for an fully

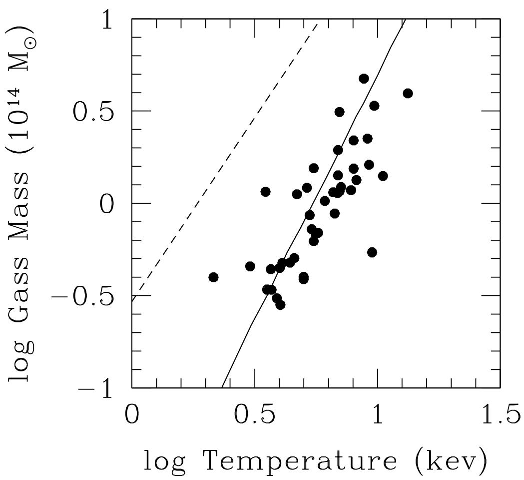

Figure 1. The gas mass-temperature relation for clusters of galaxies. The points are measured temperatures and inferred gas masses for the 42 clusters listed in Table 1. The gas mass is that obtained by extrapolating the β-model to a radius where the gas density falls to 10^{-28} g/cm^3. The dashed line is the mass-temperature relation for MOND isothermal spheres (eq. 4) and the solid curve is the gas mass-temperature relation for the two component MOND β models discussed in Section 3.

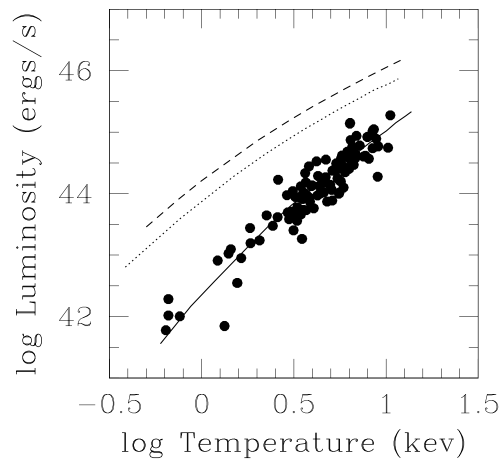

Figure 2. The X-ray luminosity-temperature relation for clusters of galaxies. The points are the clusters tabulated by Ikebe et al. scaled to h=0.7. The dashed curve is the relation for MOND isothermal spheres and the dotted curve is the relation for the near-isothermal MOND β models. The solid curve is the relation for the MOND two-component models discussed in Section 3.

ionized gas with solar abundances) and m_p is the proton mass.

This would be, in effect, the extension of the Faber-Jackson relation to clusters of galaxies (Sanders 1994). However, the observational definition of such a relation is ambiguous because the total gas mass defined by the β-model is typically divergent. If one considers the mass inside a fixed radius (Sanders 1994), or within a radius where the density falls to some fixed value (Mohr, Mathiesen & Evrard 1999), a relationship of this form (eq. 4) is observed for clusters. However, the mass at a given temperature is typically a factor of 5 to 10 below that implied by eq. 4. This is shown in Fig. 1 where the MOND mass-temperature relation for isothermal gas spheres (dashed line) is compared to the observed gas mass-temperature relation for 42 clusters listed in Table 1. Here the observed gas mass is given by the β-model (eq. 7 below) extrapolated to a radius where the gas density has fallen to about 10^{-28} g/cm^3, or 250 times the mean cosmological density of baryonic matter.

The characteristic scale of MOND isothermal gas spheres is

$$r_m \approx {\sigma_r}^2/a_o \tag{5}$$

(Sanders & McGaugh 2002). Given that the X-ray luminosity due to free-free emission is $L \propto {n_e}^2 T^{\frac{1}{2}} {r_m}^3$; then, eqs. 4 and 5 above would imply an X-ray luminosity-temperature relation of the form $L \propto T^{1.5}$. The observed luminosity-temperature relation (e.g., Ikebe et al. 2002) is significantly steeper, $L \propto T^{2.5}$, than that of MOND isothermal spheres.

To pursue this in more detail, I calculated the X-ray luminosity for MOND isothermal spheres with electron densities and temperatures similar to those of the X-ray emitting clusters of galaxies. I assume a central electron density of 0.006 cm^{-3} as being typical of clusters, and, for a given temperature, numerically integrate eqs. 1 and 2 from the center outward. The optically thin free-free radiation for the entire sphere is then calculated from the run of electron density. The resulting luminosity-temperature relation is shown by the dashed curve in Fig. 2. compared to the observations by Ikebe et al. (2002) scaled to h=0.7. In both cases, this is the radiation emitted in the band 0.2 to 2.4 keV where the X-ray flux is typically measured by satellites such as ASCA and ROSAT. Not only is the theoretical dependence shallower than observed, the predicted luminosities are an order of magnitude larger than actual clusters at the same temperature.

On the basis of these scaling relationships, it would seem that MOND isothermal spheres are not good representations of clusters of galaxies. This conclusion is reinforced when we consider the surface brightness distribution of a MOND isothermal gas sphere. Although the mass of an isotropic, isothermal sphere is effectively determined by the temperature, the detailed structure depends upon the central density; for a single temperature there is a family of solutions bounded by a limiting solution with a 1/r density cusp at the center (Milgrom 1984). The limiting MOND solution resembles the Hernquist model (Hernquist 1990) with the 1/r cusp steepening to a 1/r^4 beyond a radius r_m. Lower density spheres are characterized by a constant density core with a 1/r^4 gradient at large r; the central gas densities observed in clusters of galaxies would place these objects in this category of the sub-limiting solutions. The X-ray surface brightness distribution resulting from such a MOND isothermal sphere

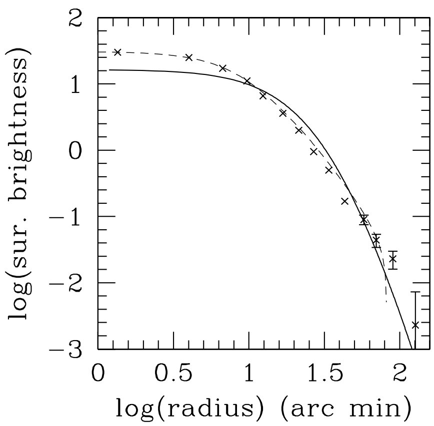

Figure 3. The radial distribution of X-ray surface brightness for a MOND isothermal sphere (T=2.5 keV) compared to observations of the Coma cluster (T=8.6 keV). The dashed curve is the β model fit (Reiprich 2000) to the observations.

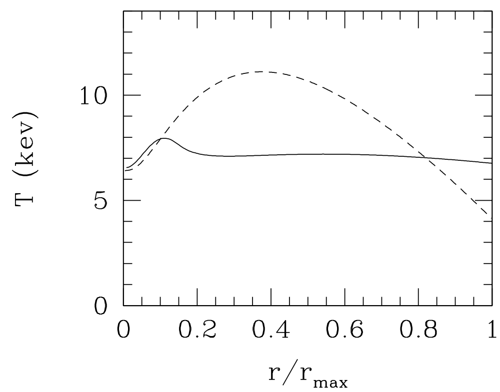

Figure 4. The projected emission-weighted temperature of a MOND β model as a function of projected radius (dashed curve). The radius is given in terms of r_{max} where the gas density has fallen to 10^{-28} g/cm^3. This is the most nearly isothermal β model. The solid curve shows the projected emission-weighted temperature for the most nearly isothermal two component model described in Section 3.

is shown in Fig. 3 compared to that observed for the Coma cluster (Briel, Henry & Böhringer 1992). These observations are well fit by the traditional β-model (Cavaliere & Fusco-Femiano 1976, Sarazin 1988),

$$I(r) = I_o\left[1+\left(\frac{r}{r_c}\right)^2\right]^{-3\beta+1/2} \qquad (6)$$

shown by the dashed curve in Fig. 3. Here we see that the MOND isothermal gas sphere provides a poor representation of the actual surface brightness distribution. Moreover, the temperature of this best fitting MOND isothermal sphere is about 2.5 keV whereas the actual temperature of the Coma cluster is in excess of 8 keV.

2.2 MOND β-models

The radial dependence of electron number density which produces the X-ray intensity distribution described by eq. 6 is

$$n_e = n_o\left[1+\left(\frac{r}{r_c}\right)^2\right]^{-1.5\beta} \qquad (7)$$

(Cavaliere & Fusco-Femiano 1976). Rather than assuming a constant temperature and solving eq. 1 for the density distribution, one may alternatively take eq. 7 as the density distribution and solve for the radial dependence of the temperature. It is necessary to specify β (typically 0.6 to 0.7 for clusters), a central electron density, n_o (again taken to be 0.006 cm^{-3}), a core radius, r_c (ranging from 50 to 300 kpc for clusters), and a central gas temperature, T_o. Then eqs. 1 and 2 may be solved for the run of temperature to an outer boundary, usually taken to be where the gas density falls to some fixed multiple of the mean cosmological density. For a given central temperature, the run of temperature is completely determined by the core radius. There is one specific value of r_c which minimizes the temperature gradients. For smaller core radii, the temperature rapidly increases toward the boundary (the models are very far from isothermal); for larger core radii, the temperature decreases to zero before the boundary is reached. Because clusters of galaxies are observed to be near isothermal, these MOND β-models with the smallest temperature variations are taken to be appropriate for clusters.

The emission-weighted temperature of this most nearly-isothermal model as a function of *projected* radius is shown by the dashed curve in Fig. 4 for $n_o = 0.006$ cm^{-3}, $\beta = 0.62$, and $T_o = 6.5$ keV. The required core radius is 436 kpc. It is evident from Fig. 4 that the model still deviates significantly from pure isothermal, with a roughly a factor of two variation around the central temperature. Moreover, compared to actual clusters, the implied core radius for this temperature is too large by a factor of two.

For these near isothermal β models, I determined the mean emission weighted temperature, the total gas mass within the cut-off radius and the X-ray luminosity within the 0.1 to 2.4 keV band. The resulting gas mass-temperature relation is almost identical to that of MOND isothermal spheres (the dashed curve in Fig. 1) and thus again much larger than the observationally inferred gas mass of X-ray emitting clusters

The luminosity-temperature relation for these MOND β models is shown in Fig. 2 by the dotted curve. Unsurprisingly, this nearly coincides with the calculated relation for MOND isothermal spheres and is clearly an equally poor description of reality. The basic problem is that, for both MOND isothermal spheres and MOND β models, the core radii are too large. It is evident that some ingredient is miss-

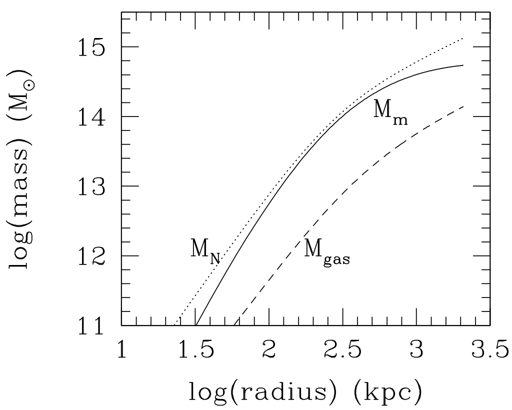

Figure 5. The accumulated mass distributions in the Coma cluster. The dotted curve is the enclosed Newtonian mass as a function of radius; the solid curve is the enclosed MOND mass; the dashed curve is the gas mass inferred from the X-ray observations

ing from these models; that an additional mass component must be added to decrease the gas core radius at a given temperature.

3 RESOLUTION OF THE PROBLEM: TWO COMPONENT MODELS

For a cluster with an observed density and temperature distribution, eqs. 1 and 2 directly yield M(r), the interior dynamical mass as a function of radius. In the Newtonian regime this is given simply by

$$M_N(r) = \frac{r}{G}\frac{kT}{fm_p}\left[\frac{d\,ln(\rho)}{d\,ln(r)} + \frac{d\,ln(T)}{d\,ln(r)}\right]_r. \tag{8}$$

With MOND, taking $\mu(x)$ to be given by eq. 2c, the dynamical mass is

$$M_m = \frac{M_N}{\sqrt{1+(a_o/a)^2}} \tag{9}$$

where a is the "observed" acceleration

$$a = \frac{1}{\rho}\frac{dp}{dr} = \frac{kT}{fm_p r}\left[\frac{d\,ln(\rho)}{d\,ln(r)} + \frac{d\,ln(T)}{d\,ln(r)}\right]_r. \tag{10}$$

Obviously, from eq. 9 in the limit of large accelerations ($a >> a_o$) the MOND dynamical mass is equivalent to the Newtonian dynamical mass.

We may apply these relations to the Coma cluster which has a density distribution well-fit by a β-model with $\beta = 0.71$, $r_c = 276$ kpc, $n_o = .0036$ (Reiprich 2001), an observed radial temperature profile (Arnaud et al. 2001) and average emission weighted temperature of 8.6 keV. The results are shown in Fig. 5 which is the cumulative Newtonian dynamical mass (dotted line), the MOND dynamical mass (solid line), and the gas mass (dashed line). Here it is evident that while the Newtonian dynamical mass continues to increase at radius of 1 Mpc, the MOND mass has essentially converged. However, the total MOND mass is still a factor of four times larger than the mass in gas alone. This discrepancy cannot be accounted for by the stellar content of the galaxies which, assuming a mass-to-light ratio of seven, amounts only to about 10^{13} $M_\odot$ within 1 Mpc (The & White 1988). This is, in fact, the discrepancy pointed out by The and White– a discrepancy which can be resolved by increasing a_o by a factor of 3 or 4 over the value required for galaxy rotation curves, or by admitting the presence of non-luminous mass which MOND does not remove.

The density of this non-luminous component is roughly constant and contained within two gas core radii. In other words the missing mass is essentially present in the inner regions of the cluster as implied in the work of Aguirre, Shaye, and Quataert (2001). This suggests that, with MOND, clusters might be described by a two component model: a gas component with a density distribution given by the β-model and a dark central component of constant density and a radius of about two times the core radius of the the gas distribution.

In Coma, the central surface density of the dark component is about $\Sigma_d = 240\ M_\odot/\mathrm{pc}^2$, which is comparable to the MOND surface density of $a_o/G \approx 700\ M_\odot\ \mathrm{pc}^2$. This is the characteristic central surface density of MOND self-gravitating isothermal systems (Milgrom 1984). Therefore, to determine scaling relations, I assume that the non-luminous component is a rigid sphere having a central surface surface density equal to that in the Coma cluster and radius twice that of the gas core radius, as in Coma. Then the constant density of the second component is

$$\rho_d = \frac{\Sigma_d}{2r_c} \tag{11}$$

and the total dark mass is

$$M_d = \frac{16\pi}{3}\Sigma_d {r_c}^2 \tag{12}$$

Evidently, with this assumption, larger clusters have a more massive dark component, but because the gas mass scales as ${r_c}^3$, the dark to gas mass decreases with increasing core radius or temperature.

The procedure followed is identical to that of the pure MOND β models described above: I assume a gas distribution described by eq. with $\beta = 0.62$, $n_o = 0.006$. Then for a given central gas temperature, I determine the core radius of the model for which the temperature gradient is minimized. The new aspect is the second component which makes its presence felt by contributing to the total cumulative mass (M(r) in eq. 2) and hence the total gravitational force. This has the effect of decreasing the core radius at a given temperature compared to the single component MOND β models. The emission-weighted temperature of as a function of projected radius is shown by the solid line in Fig. 4 again for a model with a central temperature of 6.5 keV. This is the most-nearly isothermal model, and we see that the temperature gradients are much smaller than in the the single component β-model. The total variation about the central temperature is less than 40%. In other words, *isothermal β models require, in the context of MOND, this second central component with roughly constant density.*

The gas-mass-temperature relation of such models is shown by the solid line in Fig. 1 which is evidently consistent with the observations. The luminosity-temperature relation for these two component cluster models is shown by the solid line in Fig. 2. These models provide a reasonable description of the observed relation. This is due to the fact that, in the low temperature systems, a relatively larger fraction of the mass is not in gaseous form. It is also not in the form of luminous material in galaxies as the implied mass-to-light ratios would be too large. Agreement of MOND with the cluster scaling relations is achieved at the expense of adding unseen matter.

4 MODELING INDIVIDUAL CLUSTERS

Individual clusters may be described by such two component models. Given the parameters of a β model fit to the X-ray emission from an specific cluster (i.e., β, n_o, and r_c), and a characteristic temperature for the entire cluster, I determine, via eqs. 1 and 2, the central temperature, T_o, and the density (or surface density, related by eq. 11) of the dark component which yields the observed emission weighted temperature and the observed core radius, r_c of the β model. In all cases the dark component is assumed to extend to $2r_c$. Uniqueness is ensured by requiring that the temperature gradients be minimized– i.e., these are again the most nearly isothermal models. The fitting parameter is the density of the central dark component which yields the total dark mass via eq. 12 above.

Table 1 lists the clusters which have been modeled in this way along with the the parameters of the β-model fit and the mean temperature, all from the compilation of Reiprich (2001) with cluster properties scaled to h=0.7. The the required central surface density of the dark component, Σ_d, is given along with the total gas mass (out to the cutoff radius) and the total mass of the dark component. The enclosed Newtonian dynamical mass is also given.

This sample was chosen to include a number of objects with significant inferred cooling flows (such as Abel 1689 and 2029). These clusters are characterized by relatively small core radii ($r_c < 100$ kpc) and large central electron densities ($n_o > 0.01$ cm^{-3}). The sample also includes clusters at the other extreme– large core radii ($r_c > 250$ kpc) and low central electron densities ($n_o < 0.005$ cm^{-3}) with no inferred cooling flow (such as Abel 119 and 2256).

In general, MOND reduces the classical Newtonian discrepancy for clusters of galaxies. For this sample, with MOND the ratio of the total dark mass to the gas mass is 1.60 $\pm$ 1.7. The Newtonian dark mass-to-gas mass ratio is 7.14 $\pm$ 2.7. While this is a significant reduction, it is also clear that MOND does not fully resolve the mass discrepancy in clusters. Moreover, it should be noted that the ratio of masses of the dark-to-hot gas components in the central two core radii (where the additional component is required) can be as large as 10, as was also pointed out by Aguirre, Schaye & Quataert (2001). This rigid component cannot be stars of a normal population because the mass-to-light ratio within two core radii would still be, on average, in excess of about 50. The principle question is whether or not this dark component is a fundamental problem or can be accommodated in the context of MOND.

It could be that a high M/L population of low mass stars or sub-stellar objects is deposited in the central regions of clusters as a result of cooling flows. Fig. 6 is a plot of the total mass in the dark component vs. the cooling rate as estimated by White, Jones & Forman (1997). The solid points are those clusters with a large discrepancy: $M_d/M_g > 1.5$. The crosses are clusters with a smaller discrepancy: $M_d/M_g < 1.5$. We see that there is no obvious correlation between the total mass of the dark component and the cooling rate– especially for those clusters with the largest discrepancy. On the other hand, in Fig. 7 we see a plot of the surface density of the dark component vs. the mass deposition rate. Here there does appear to be a correlation. This is because those clusters with the highest central dark matter densities are not the clusters with the largest mass discrepancies.

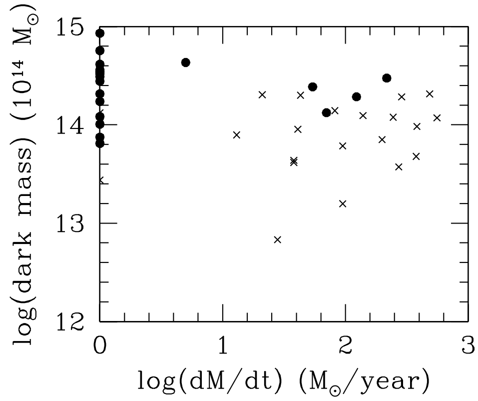

Figure 6. A log-log plot of the fitted dark mass ($10^{14}M_\odot$) vs. the cooling mass inflow rate inferred from central gas densities and temperatures (White, Jones & Forman 1997). The crosses are those clusters with a small mass discrepancy ($M_d/M_g < 1.5$), and the solid points are the objects with large discrepancies ($M_d/M_g > 1.5$).

It is unclear if this apparent correlation between surface density and mass deposition rate is significant. The mass deposition is not actually observed but calculated from the central gas density. Those clusters with large inferred cooling flows are clusters with high central gas densities and small core radii. But it precisely these clusters which require a large surface density of dark matter to produce the small core radius. While it may be the case that cooling flows contribute to the dark component of those clusters with the largest central density of dark matter, it is also evident from Fig. 6 that this cannot be the explanation for the discrepancy in clusters with the largest dark mass problem. These tend to be the clusters with low central gas densities and low inferred dark matter densities– but large core radii. In the next section, I consider another possibility: particle dark matter in the form of massive neutrinos.

Table 1. Two component model fits to observed clusters

Cluster	T	β	r_c	n_o	Σ_d	$\dot{M}_c$	M_g	M_d	M_N
	keV		kpc	$10^{-3}cm^{-3}$	$100M_\odot/pc^2$	$M_\odot$/year	$10^{14}M_\odot$	$10^{14}M_\odot$	$10^{14}M_\odot$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
A85	6.9	0.532	58.1	20.4	12.5	198	1.94	0.704	10.9
A119	5.6	0.675	359.	1.25	0.8	0	0.679	1.73	7.27
A262	2.15	0.443	30.	9.57	4.5	27	.397	0.0678	2.83
A399	7.0	0.713	320.	2.54	1.62	0	1.17	2.77	10.7
A401	8.0	0.613	175.	6.72	3.9	42	2.19	1.99	13.8
A576	4.02	0.825	283.	1.88	0.99	69	0.275	1.35	4.17
A754	9.5	0.698	171.	5.21	6.1	216	0.589	2.94	8.09
A1367	3.55	0.695	273.	1.51	.6	0	0.35	.738	3.82
A1644	4.7	0.579	214.	2.60	1.03	12	1.12	0.789	7.64
A1651	6.1	0.693	129.	9.26	4.45	138	1.05	1.22	7.76
A1650	6.7	0.704	201.	5.08	2.85	122	0.893	1.92	8.10
A1656	8.21	0.705	275.	3.64	2.6	0	1.36	3.27	12.2
A1689	9.23	0.690	116.	20.2	9.15	484	1.60	2.08	11.0
A1736	3.5	0.542	267.	1.51	0.23	0	1.16	0.275	6.87
A1795	7.8	0.596	56.5	30.3	18.	381	1.18	0.96	7.97
A1914	10.53	0.751	167.	13.2	7.5		1.43	3.48	12.1
A2029	9.1	0.582	59.3	34.2	20.0	556	2.21	1.21	12.5
A2052	3.03	0.526	26.4	31.7	13.5	94	0.459	0.154	3.24
A2063	3.68	0.561	78.6	7.42	4.0	37	0.447	0.407	3.74
A2065	5.5	1.162	496.	2.36	1.38	0	0.619	5.69	10.5
A2142	9.7	0.591	110.	16.3	9.5	286	3.35	1.95	18.0
A2163	13.29	0.796	371.	6.17	3.7	0	3.93	8.54	27.3
A2199	4.1	0.655	99.3	9.84	3.7	94	0.475	0.612	4.08
A2244	7.1	0.607	90.	14.2	8.8	244	1.19	1.23	8.51
A2255	6.87	0.797	424.	1.95	1.2	0	1.12	3.63	11.5
A2256	6.6	0.914	419.	3.1	1.4	0	1.19	4.08	11.7
A2319	8.8	0.591	204.	6.96	2.9	20	4.80	2.02	23.2
A2597	4.4	0.633	41.4	43.2	13.	271	0.476	0.374	3.59
A2634	3.7	0.640	261.	1.27	.566	0	0.339	0.648	3.69
A3112	5.3	0.576	43.6	33.1	15.	376	0.869	0.471	5.75
A3266	8.0	0.796	403.	2.74	1.58	4	1.57	4.21	14.0
A3376	4.0	1.054	539.	1.21	0.5	53	0.444	2.44	6.50
A3391	5.4	0.579	167.	3.04	2.24	0	0.706	1.06	6.32
A3395n	5.0	0.981	478.	1.21	0.89		0.384	5.02	7.36
A3395s	5.0	0.964	431.	1.51	.99		0.405	3.07	7.14
A3530	3.89	0.773	301.	1.51	8.0		0.307	1.21	4.27
A3532	4.58	0.653	201.	2.74	1.5	0	0.507	1.01	5.09
A3558	5.5	0.580	160.	5.17	2.1	40	1.56	0.894	9.60
A3562	5.16	0.472	70.7	6.69	5.2	37	1.22	0.432	7.64
A3571	6.9	0.613	129.	8.75	5.0	81	1.40	1.41	9.70
A3667	7.0	0.541	199.	4.01	2.0	0	3.13	1.32	16.7
A3921	5.73	0.762	237.	4.06	2.2		0.697	2.07	7.30

All β-model parameters are from the compilation of Reiprich (2001). In columns 8 and 10 the masses are those enclosed within a radius (typically 1 to 3 Mpc) where the gas density has fallen to 250 times its mean cosmic value (assuming $\Omega_b = 0.04$). In column 7 a blank entry means that there is no estimate for the cooling mass inflow rate.

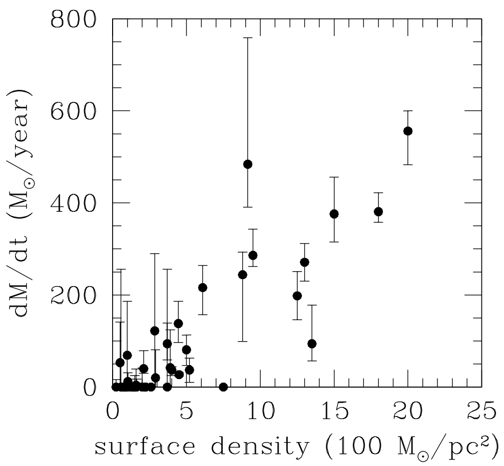

Figure 7. The inferred cooling mass inflow rate in the sample clusters (as in Fig. 6) vs. the central surface density of the dark component in units of 100 $M_\odot/pc^2$.

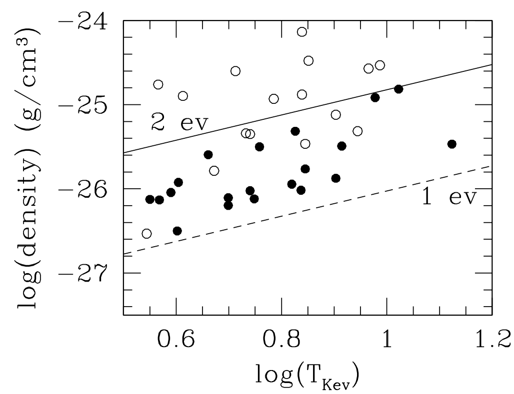

Figure 8. A log-log plot of the fitted central density of the dark component in the sample clusters vs. the temperature. The solid line is the relation between the maximum neutrino density and the temperature imposed by phase space constraints for three neutrino types all with mass 2 ev (eq. 13). The dashed line is the same when the neutrino mass is taken to be 1 ev. The solid points are those clusters with a large discrepancy ($M_d/M_g > 1.5$).

5 NEUTRINOS AS CLUSTER DARK MATTER

The most well-motivated form of particle dark matter consists of neutrinos with finite mass. Aspects of the observed fluxes of atmospheric and solar neutrinos provide strong evidence for neutrino oscillations and hence non-zero neutrino masses (Gonzales-Garcia & Nir 2002). The fact that the number density of neutrinos produced in the early universe is comparable to that of photons then implies that there is a universal dark matter sea of neutrinos. The contribution to cosmological mass density would be $\Omega_\nu h^2 = \sum m_{\nu_i}/94$ ev where the sum is over neutrino types.

The neutrino oscillation experiments do not provide information on the actual masses of neutrino species but on the square of the mass differences. These are small, such that, the largest mass difference, suggested by the atmospheric oscillations, is $\Delta m \approx 0.05$ ev. If $m_\nu \approx \Delta m$ then the three active neutrino types would have no significant cosmological mass density ($\approx 10^{-3}$) and could not contribute to the mass budget of any bound astronomical object. But another possibility is that $m_\nu >> \Delta m$ and that the masses of all three active types are nearly equal. In this case an upper limit to the masses is provided by an experimental limit on the mass of the electron neutrino, i.e. 2.2 ev at 90% c.l. (Groom et al. 2000) If it were the case that the electron neutrino mass were near 2 ev, then neutrinos would constitute a significant fraction of the cosmic density ($\Omega_\nu \approx 0.13$ for h=0.7).

However, neutrinos of this mass could not contribute to the mass budget of galaxies. This follows from a classic argument by Tremaine & Gunn (1979) based upon conservation of the phase space density of the neutrino fluid. Relativistic neutrinos are created with a maximum phase space density of $(2\pi\hbar)^{-3}$ per type including anti-neutrinos (this is a factor of two less than the absolute limit implied by quantum mechanical degeneracy). In subsequent evolution of the neutrino fluid involving gravitational instability and collapse, the final phase space density cannot exceed this value. This provides a relation between the final density of neutrino dark matter and the velocity dispersion of the system; with three types

$$\rho_\mu \leq (4.8\times 10^{-27})\Big(\frac{m_\nu}{2\,ev}\Big)^4 (T_{keV})^{\frac{3}{2}}\,\mathrm{g/cm^3} \qquad (13)$$

Equivalently, for virialized systems, this may be written as a relation between the effective core radius of a dark halo and its velocity dispersion; this is, roughly,

$$r_d \geq 0.5\Big(\frac{2\,ev}{m_\nu}\Big)^2\Big(\frac{1000\,\mathrm{km/s}}{\sigma_r}\Big)^{\frac{1}{2}} Mpc \approx 0.7\Big(\frac{2\,ev}{m_\nu}\Big)^2 (T_{keV})^{-\frac{1}{4}}\,\mathrm{Mpc} \qquad (14)$$

Clearly for objects with the required velocity dispersion of galaxy halos ($25\,\mathrm{km/s} < \sigma_r < 200\,\mathrm{km/s}$), neutrinos in the mass range of one to two ev could not possible cluster on sub-Mpc scales. However, it obviously would be possible for such neutrinos to contribute to the mass budget of large clusters.

Fig. 8 shows the the fitted matter density of the dark cores of the clusters of galaxies listed in Table 1 vs. the gas temperature of the clusters. The solid points are the clusters with significant mass discrepancies ($M_d/M_g > 1.5$) and the open points the clusters with lower discrepancies ($M_d/M_g < 1.5$). The solid and dashed lines show the relation between maximum neutrino density and the temperature implied by eq. 13, for neutrinos of 2 ev and 1 ev respectively. For neutrinos of 2 ev, this relation appears to form an upper envelope for the clusters with the largest discrepancies. However, due to the extreme sensitivity of this density

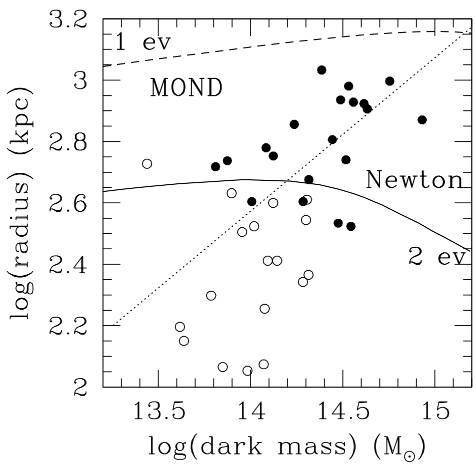

Figure 9. The solid curve is radius-mass relation for self-gravitating objects supported by degenerate neutrino pressure, assuming three neutrino types of mass 2 ev. The dashed line is the same when the neutrino mass is taken to be 1 ev. The dotted line separates the Newtonian and MOND regimes and the points are relevant to the dark matter cores of the sample clusters. The closed points are those clusters with significant discrepancies as before.

limit on neutrino mass (eq. 13), neutrinos with mass as low as 1 ev could not comprise the dark component.

I have calculated the structure of a self-gravitating system supported by the degenerate pressure of neutrinos (a neutrino star). In the non-relativistic case there is a pressure-density relation of the form $p_\nu = K{\rho_\nu}^{5/3}$ where $K = 5.5 \times 10^{32}(m_\nu/2\,ev)^{-\frac{8}{3}}$, and density is in g/cm^3. Inserting this relation in eq. 1, I determine the run of density in a system with a given central velocity dispersion. Such objects (n=1.5 polytropes) have nearly constant density out to a finite radius where the density drops to zero. As for white dwarfs, there is a mass-radius relation. This is shown by the solid curve ($m_\nu = 2$ ev) and the dashed curve ($m_\nu = 1$ ev) in Fig. 9 where I have assumed that the central velocity dispersion of the neutrinos is equal to the gas velocity dispersion. The solid points show the mass and radius of the dark matter cores for those clusters with significant discrepancies ($M_d/M_g > 1.5$) and the open points those with small discrepancies. The dotted line separates the MOND and Newtonian regime where we see that the mass-radius relation has different forms: $r \propto M^{1/12}$ in the MOND regime and $r \propto M^{-1/3}$ in the Newtonian regime. It is of interest that for $M_\nu = 2$ ev the range of radii and masses correspond in magnitude to the dark cores required in clusters. Moreover, we see that those clusters with large discrepancies lie generally above the mass-radius relation as would be expected for neutrino clusters with density less than or equal to that given by eq. 13. Further, not even neutrinos of 2 ev apparently could contribute to the mass budget of those objects with low discrepancies.

That neutrinos in the range of one to two ev make up the missing mass in clusters is a provocative possibility, and one which is within reach of experimental verification. The accelerator limits on the electron mass can soon be pushed to within a few tenths of an electron volt; if there is no positive detection, then active neutrinos cannot supply the dark mass of clusters.

6 CONCLUSIONS

What one may call, generically, "the dark matter problem" first became evident with radial velocity studies of rich clusters of galaxies (Zwicky 1933). The discrepancy between the visible and Newtonian dynamical mass, quantified then in terms of mass-to-light ratio, was more than a factor of 100. With the advent of X-ray observatories and the detection of hot gas in clusters, this discrepancy between dynamical and detectable mass was reduced to a factor of 10. Modified Newtonian dynamics reduces the discrepancy further, to a factor of 2 to 3, but it is clear that a discrepancy remains which cannot be explained by detected gaseous or luminous mass. It is also apparent that while the missing mass is primarily in the inner regions of clusters, it does extend beyond the core radius as defined by the gas density distribution.

How serious is this remaining mass discrepancy for MOND? Formally speaking, it does not constitute a falsification. If the dynamical mass predicted by MOND were generally less than the detected mass in stars and gas, then it would be a definite falsification, but this is not the case. More mass can always be found (as in the case of the hot gas), but it is difficult to make observed mass disappear.

One might reasonably argue that the implied presence of undetected mass runs against the spirit of MOND, which, in the view of most people, was suggested primarily as a replacement for dark matter. But more generally, MOND should not be viewed simply as an alternative to dark matter; the systematic appearance of the mass discrepancy in astronomical systems with low internal accelerations is an indication that Newtonian dynamics or gravity may break down in this limit. MOND primarily addresses this issue: is physics in the low-acceleration regime Newtonian? The success of MOND in explaining the scaling properties and observed rotation curves of galaxies suggests that it may not be. MOND does not rest upon the principle that there is no undetected or dark matter. Indeed, comparing the density of luminous matter to the baryonic content of the universe implied by considerations of primordial nucleosynthesis, one can only conclude that there is, as yet, undetected baryonic matter, probably in the form of diffuse gas in the intergalactic medium. Moreover, it is virtually certain that particle dark matter exists in the form of neutrinos; only its contribution to the total mass density of the Universe is unclear.

MOND would be incompatible with the wide-spread existence of dark matter which clusters on the scale of galaxies– cold dark matter. But MOND is not inconsistent with hot dark matter such as 2 ev neutrinos, which can only aggregate on the scale of clusters of galaxies– indeed, I have presented evidence that this may be the case. Neutrinos, as particle dark matter candidates, are unquestionably well-motivated, both from a theoretical point-of-view (they definitely exist) and from an experimental point-of-view (they have mass). No conjectured CDM particle shares these advantages. While I do not wish to state that the dark matter in clusters is definitely in the form of 2 ev neutrinos (there is more than enough remaining baryonic matter to make up the missing mass), there are indications that point this way. The largest discrepancies are found in the clusters with the largest core radii as would be the case with neutrinos. The indicated densities of dark matter are comparable to the maximum possible density of 2 ev neutrinos. The radius-mass relationship for self-gravitating degenerate neutrino objects forms an envelope for those clusters with large discrepancies.

The fact remains that there exists an algorithm, MOND, which allows galaxy rotation curves to be predicted in detail from the observed distribution of matter, and it is for these systems that the kinematic observations are most precise. This fact challenges the current CDM paradigm, and demands explanation if dark matter lies behind the discrepancy. The factor two remaining discrepancy in clusters is less challenging for MOND, particularly given that MOND makes no claims about the full material content of the Universe.

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REFERENCES

Aguirre A., Schaye J., Quataert E., 2001, ApJ, 561, 520
Arnaud M. et al., 2001, A&A, 365, L67
Briel U.G., Henry J.P., Böhringer H., 1992, A&A, 259, L31
Cavaliere A.L., Fusco-Femiano R., 1976, A&A, 49, 137
Gerbal D., Durret F., Lachièze-Rey M., Lima-Neto G., 1992, A&A, 262, 395
Gonzalez-Garcia M.C., Nir Y., 2002, hep-th/0202058
Groom D.E. et al., 2000, Eur.Phys.J., C15, 1
Hernquist L., 1990, ApJ, 356, 359
Ikebe I., Reiprich T.H., Böhringer H., Tanaka Y., Kitayama T., 2002, A&A, 383, 773
Milgrom M., 1983a, ApJ, 270, 365
Milgrom M., 1983b, ApJ, 270, 371
Milgrom M., 1983c, ApJ, 270, 384
Milgrom M., 1984, ApJ, 287, 571
Milgrom M., 1997, ApJ, 478, 7
Milgrom M., 1998, ApJ, 496, L89
Mohr J.J., Mathiesen B., Evrard A.E., 1999, ApJ, 517, 627
Reiprich T.H., 2001, PhD. Dissertation, *Cosmological Implications and Physical Properties of an X-Ray Flux-Limited Sample of Galaxy Clusters*, Ludwig-Maximilians-Universität München
Sanders R.H., 1994, A&A, 284, L31
Sanders R.H., 1999, ApJ, 512, L23
Sanders R.H., McGaugh S.S., 2002, Ann.Rev.A&A, 40, 263
Sarazin C.L., 1988, *X-ray Emissions from Clusters of Galaxies*, Cambridge University Press, Cambridge
The L.S., White S.D.M., 1988, AJ, 95, 1642
Tremaine S., Gunn J.E., 1979, Phys.Rev.Lett., 42, 408
White D.A., Jones C., Forman W., 1997, MNRAS, 292, 419
Zwicky F., 1933, Helv.Phys.Acta, 6, 110